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# Industrial Process Control

## MDP 454

If you have a smart project, you can say "I'm an engineer" ”

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## Lecture 8

Staff boarder

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# Industrial Process Control

## MDP 454

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- **Lecture aims:**
  - Be familiar with the design of lead and lag compensators using Bode plot methods
  - Understand the design of a control system is concerned with the arrangement, or the plan, of the system structure and the selection of suitable components and parameters

# Bode Plots

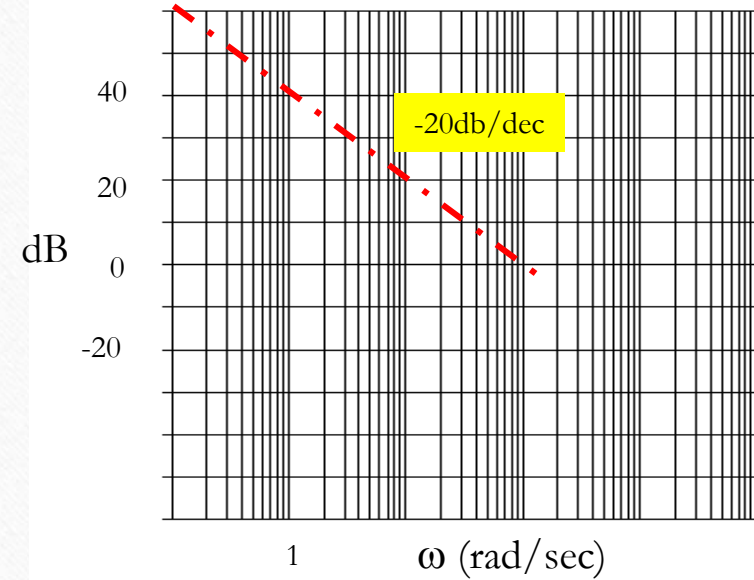
Example : Given the transfer function. Plot the Bode magnitude.

$$G(s) = \frac{100(1 + s/10)}{s(1 + s/100)^2}$$

Consider first only the two terms of

Which, when expressed in dB, are;  $20\log 100 - 20 \log \omega$ .  
This is plotted below.

The  $20\log 100 - 20 \log \omega$  is  
a tentative line we use  
until we encounter the  
first pole(s) or zero(s)  
not at the origin.





# Bode Plots

Example :

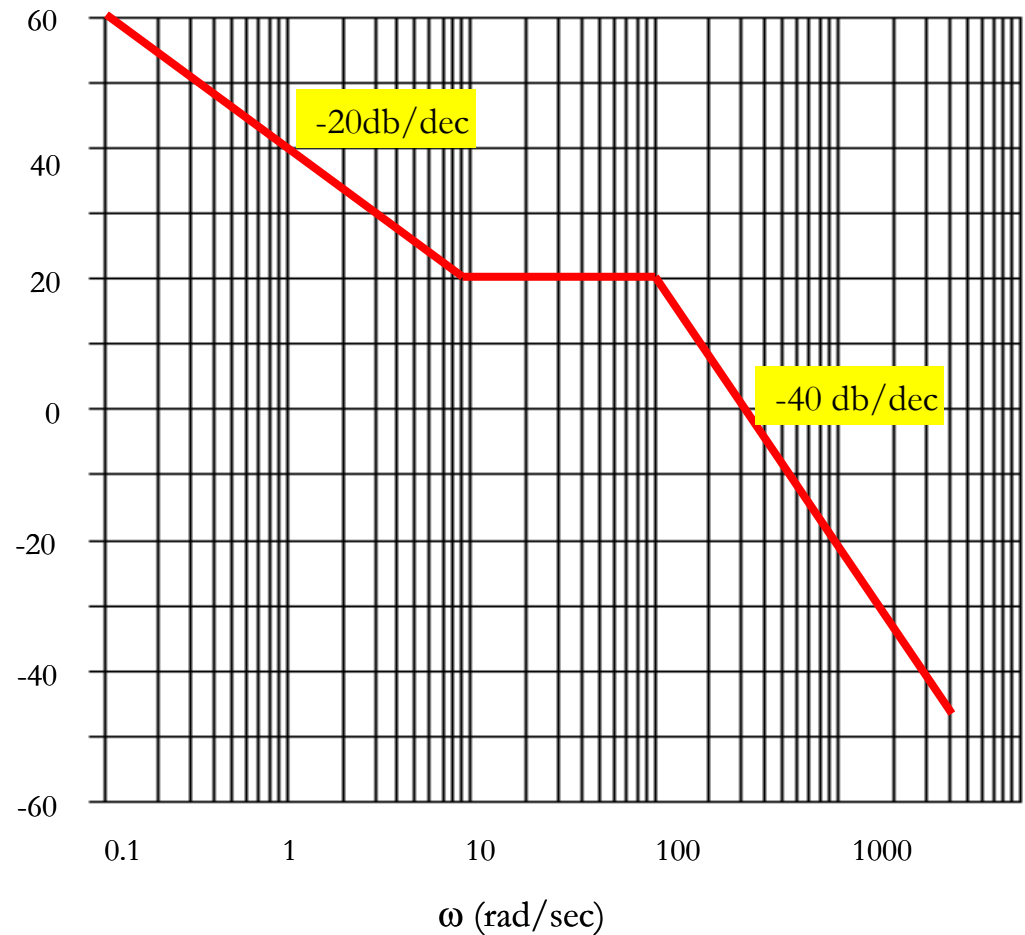
The completed plot is shown below.

$$G(s) = \frac{100(1 + s/10)}{s(1 + s/100)^2}$$

dB Mag

**Phase**

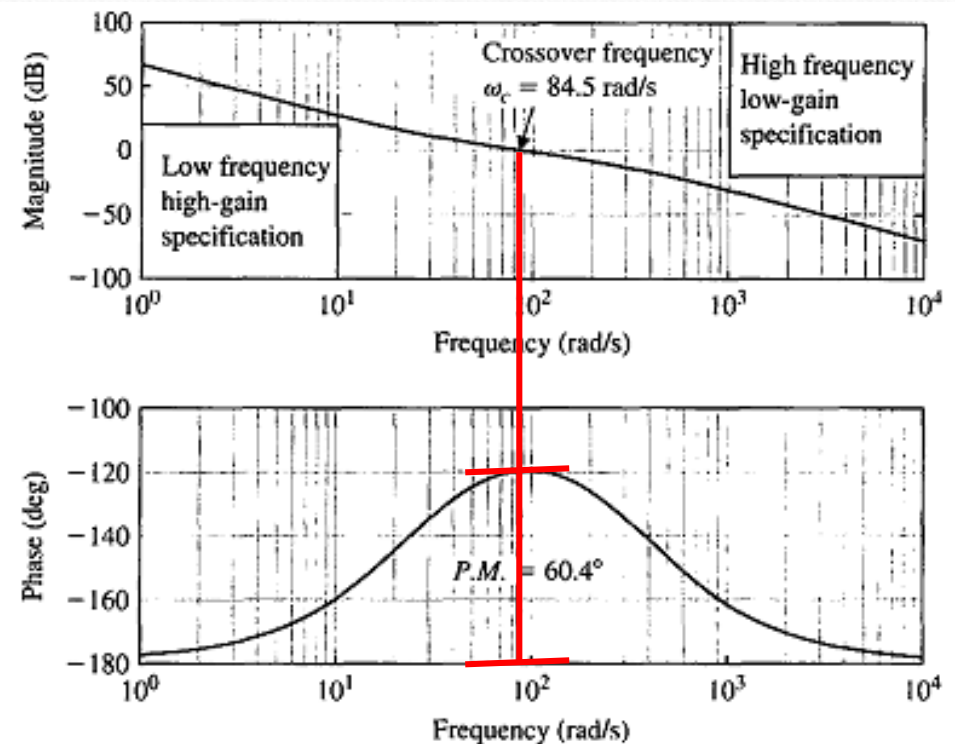
$$\angle G(j\omega) = \tan^{-1}(\omega/10) - \tan^{-1}(\infty) - \tan^{-1}(\omega/100)$$



Phase (deg)

# Stability in Frequency Response

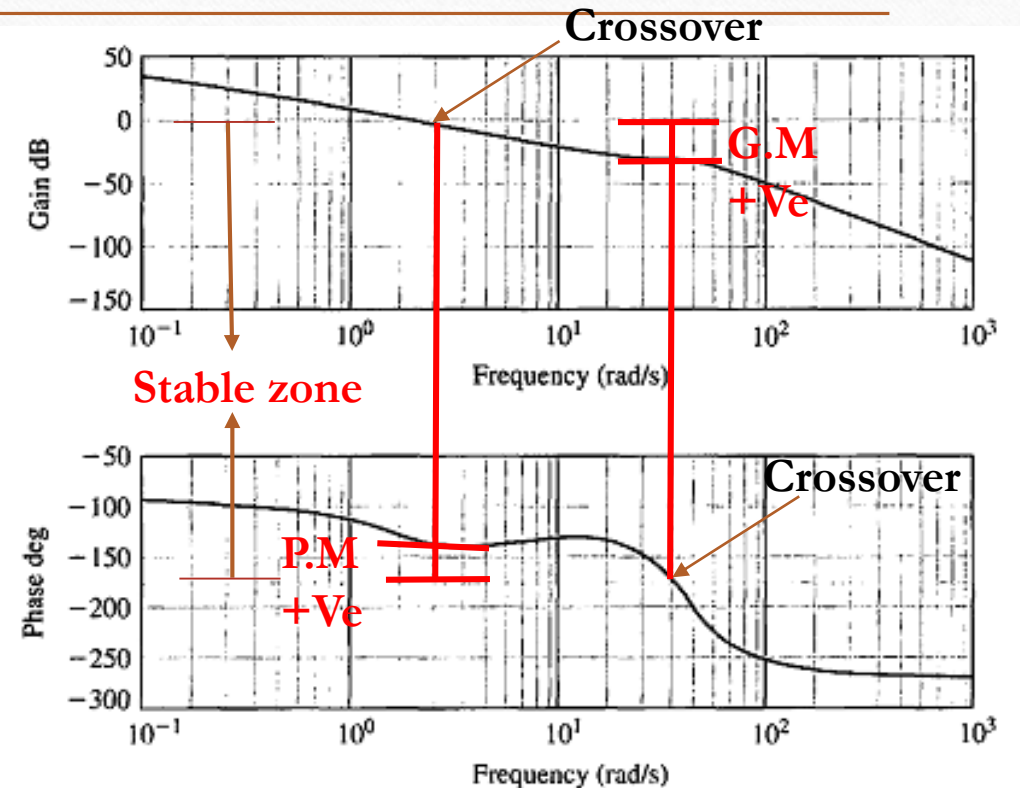
- G.M = +Ve if below zero of magnitude
- P.M = +Ve if above -180 of phase
- Stable if G.M & P.M are +ve
- Unstable if G.M & P.M are -ve
- Critical stability if G.M & P.M are in opposite signs





# Stability in Frequency Response

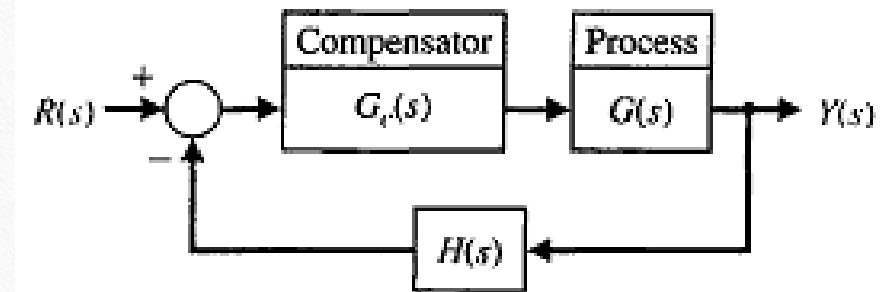
- G.M = +Ve if below zero of magnitude
- P.M = +Ve if above -180 of phase
- Stable if G.M & P.M are +ve
- Unstable if G.M & P.M are -ve
- Critical stability if G.M & P.M are in opposite signs



# Design of Compensator

- A compensator is an additional component or circuit that is inserted into a control system to compensate for a deficient performance

The transfer function of a compensator is designated as  $G_c(s) = E_o(s)/E_i(s)$ , and the compensator can be placed in a suitable location within the structure of the system



# Design of Compensator

<https://www.youtube.com/watch?v=xLhvil5sDcU>

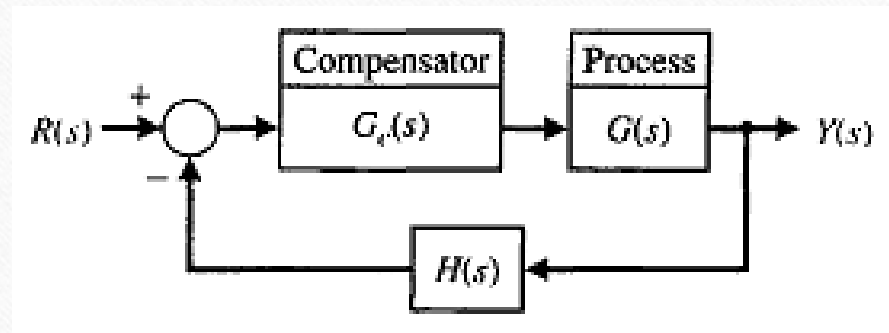


# Phase-lead network

- The design problem then becomes the selection of  $z, p$ , and  $K$  in order to provide a suitable performance.
- The **first-order compensator** with the transfer function

$$G_c(s) = \frac{K(s + z)}{s + p}$$

A compensator  $G_c(s)$  is used with a process  $G(s)$  so that the overall loop gain can be set to satisfy the steady-state error requirement, and then  $G_c(s)$  is used to adjust the system dynamics



# Phase-lead network

## lead compensator for a system

- Steady-state error for a ramp input  $K_v = \frac{A}{e_{ss}}$

The velocity constant of a type-one uncompensated

$$K_v = \lim_{s \rightarrow 0} s\{G(s)\},$$

$$G(s) = \frac{K \prod_{i=1}^M (s + z_i)}{s \prod_{j=1}^n (s + p_j)},$$

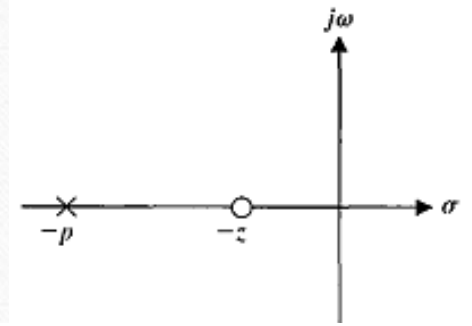
$$K_v = \frac{K \prod_{i=1}^M z_i}{\prod_{j=1}^n p_j}.$$

# Phase-lead network

- If  $|p| \gg |z|$ , and the zero occurred at the origin of the s-plane

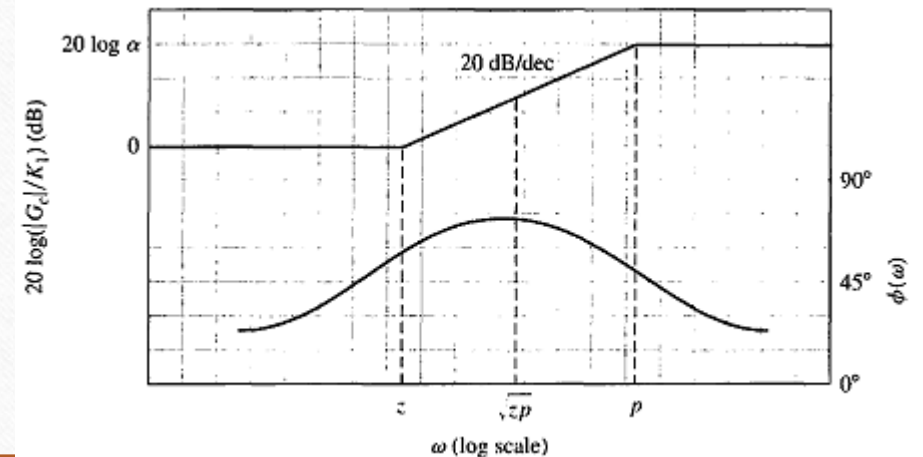
$$G_c(s) \approx \frac{K}{p}s.$$

- we would have a differentiator, and the frequency characteristic and a phase angle of  $+90^\circ$



$$G_c(j\omega) = \frac{K(j\omega + z)}{j\omega + p} = \frac{(Kz/p)[j(\omega/z) + 1]}{j(\omega/p) + 1} = \frac{K_1(1 + j\omega\alpha\tau)}{1 + j\omega\tau}$$

where  $t = 1/p$ ,  $p = \alpha z$ , and  $K_1 = K/\alpha$ .





# Phase-lead network

The phase-lead compensation transfer function can be obtained with the network shown in Figure

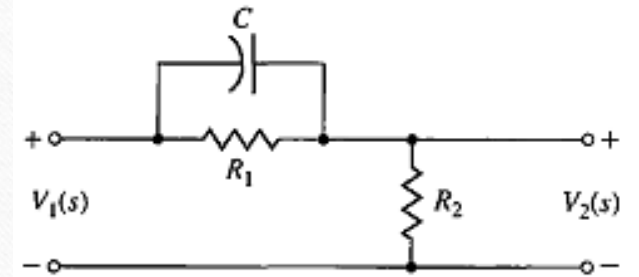
$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \frac{R_1/(Cs)}{R_1 + 1/(Cs)}} = \frac{R_2}{R_1 + R_2} \frac{R_1 Cs + 1}{[R_1 R_2 / (R_1 + R_2)] Cs + 1}$$

we let  $\tau = \frac{R_1 R_2}{R_1 + R_2} C$  and  $\alpha = \frac{R_1 + R_2}{R_2}$ ,

we obtain the **phase-lead compensation** transfer function

The maximum value of the phase lead occurs at a frequency  $\omega_m$ ,

The maximum phase lead occurs halfway between the pole and zero frequencies



$$G_c(s) = \frac{1 + \alpha \tau s}{\alpha(1 + \tau s)}$$

$$\omega_m = \sqrt{z p} = \frac{1}{\tau \sqrt{\alpha}}$$

# Phase-lead network

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$$G_c(j\omega) = \frac{K(j\omega + z)}{j\omega + p} = \frac{(Kz/p)[j(\omega/z) + 1]}{j(\omega/p) + 1} = \frac{K_1(1 + j\omega\alpha\tau)}{1 + j\omega\tau}$$

The angle of the frequency characteristic is

$$\phi(\omega) = \tan^{-1}(\alpha\omega\tau) - \tan^{-1}(\omega\tau)$$

Because the zero occurs first on the frequency axis, we obtain a phase-lead characteristic. The slope of the asymptotic magnitude curve is +20 dB/decade

# Phase-lead network

To obtain an equation for the maximum phase-lead angle

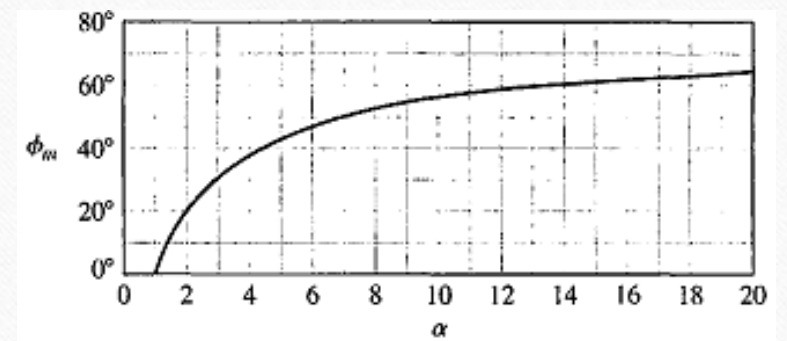
$$\phi = \tan^{-1} \frac{\alpha\omega\tau - \omega\tau}{1 + (\omega\tau)^2\alpha}$$

substituting the frequency for the maximum phase angle,  $\omega_m$

$$\tan \phi_m = \frac{\alpha/\sqrt{\alpha} - 1/\sqrt{\alpha}}{1 + 1} = \frac{\alpha - 1}{2\sqrt{\alpha}}$$

$$\sin \phi_m = \frac{\alpha - 1}{\alpha + 1}$$

There are practical limitations on the maximum value of  $\alpha$  that we should attempt to obtain. if we required a maximum angle greater than  $70^\circ$ , two cascade compensation networks would be used.





# Phase-lead network

**Determine the compensation network by completing the following steps:**

1. Evaluate the **uncompensated** system phase margin when the error constants are satisfied.
2. Allowing for a small amount of safety, **determine** the necessary additional **phase lead**  $\phi_m$ .
3. **Evaluate**  $\alpha$  from Equation.

$$\sin \phi_m = \frac{\alpha - 1}{\alpha + 1}$$

4. Evaluate  **$10 \log \alpha$**  and determine the frequency where the uncompensated magnitude curve is equal to  $-10 \log \alpha$  dB. Because the compensation network provides a gain of  **$10 \log \alpha$  at  $\omega_m$** , this frequency is the new 0-dB crossover frequency and  $\omega_m$  simultaneously.
5. **Calculate** the pole  $p = \omega_m \sqrt{\alpha}$  and  $z = p/\alpha$ .
6. **Draw** the **compensated** frequency response, **check** the resulting phase margin, and repeat the steps if necessary. **Finally, for an acceptable design, raise the gain of the amplifier in order to account for the attenuation ( $1/\alpha$ ).**

# Phase-lead network

Determine the compensation network by completing the following steps:

**EXAMPLE 10.2 A lead compensator for a second-order system**

A unity feedback control system has a loop transfer function

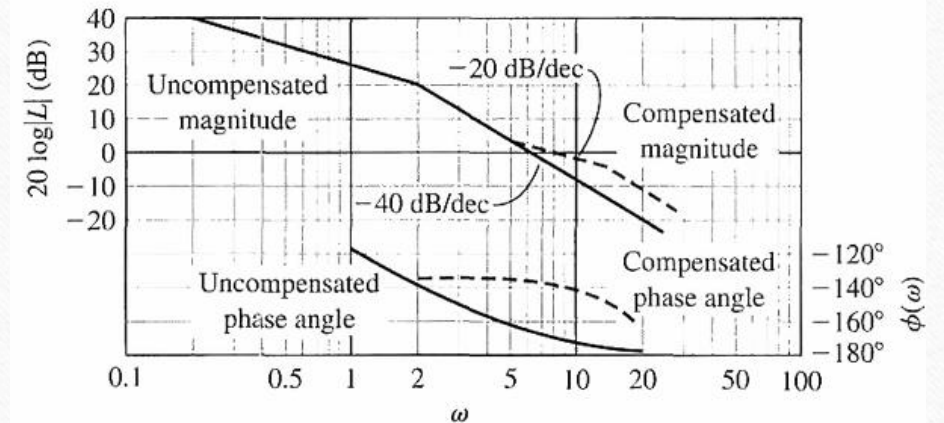
$$L(s) = \frac{K}{s(s + 2)}, \quad (10.21)$$

where  $L(s) = G_c(s)G(s)$  and  $H(s) = 1$ . We want to have a steady-state error for a ramp input equal to 5% of the velocity of the ramp. Therefore, we require that

$$K_v = \frac{A}{e_{ss}} = \frac{A}{0.05A} = 20. \quad (10.22)$$

Furthermore, we desire that the phase margin of the system be at least  $45^\circ$ . The first step is to plot the Bode diagram of the uncompensated transfer function

$$G(j\omega) = \frac{K_v}{j\omega(0.5j\omega + 1)} = \frac{20}{j\omega(0.5j\omega + 1)}, \quad (10.23)$$



$$20 \log 20 = 26.02$$

at  $\omega = 1$



# Phase-lead network

Determine the compensation network by completing the following steps:

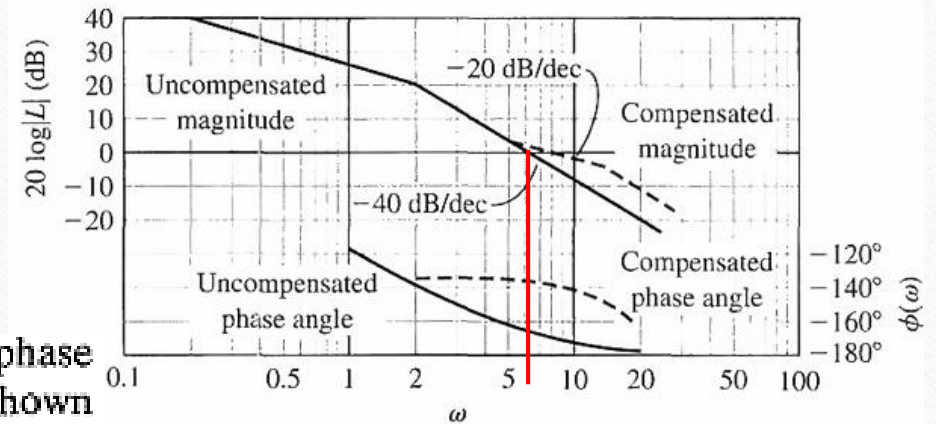
as shown in Figure 10.10(a). The frequency at which the magnitude curve crosses the 0-dB line is 6.2 rad/s, and the phase margin at this frequency is determined readily from the equation of the phase of  $G(j\omega)$ , which is

$$\angle G(j\omega) = \phi(\omega) = -90^\circ - \tan^{-1}(0.5\omega). \quad (10.24)$$

At the crossover frequency  $\omega = \omega_c = 6.2$  rad/s, we have

$$\phi(\omega) = -162^\circ, \quad (10.25)$$

and therefore the phase margin is  $18^\circ$ . Using Equation (10.24) to evaluate the phase margin is often easier than drawing the complete phase-angle curve, which is shown in Figure 10.10(a). Thus, we need to add a phase-lead network so that the phase margin is raised to  $45^\circ$  at the new crossover (0-dB) frequency. Because the compensation crossover frequency is greater than the uncompensated crossover frequency, the phase lag of the uncompensated system is also greater. We shall account for this additional phase lag by attempting to obtain a maximum phase lead of  $45^\circ - 18^\circ = 27^\circ$ , plus a small increment (10%) of phase lead to account for the added lag. Thus, we will design a compensation network with a maximum phase lead equal to  $27^\circ + 3^\circ = 30^\circ$ . Then, calculating  $\alpha$ , we obtain





# Phase-lead network

Determine the compensation network by completing the following steps:

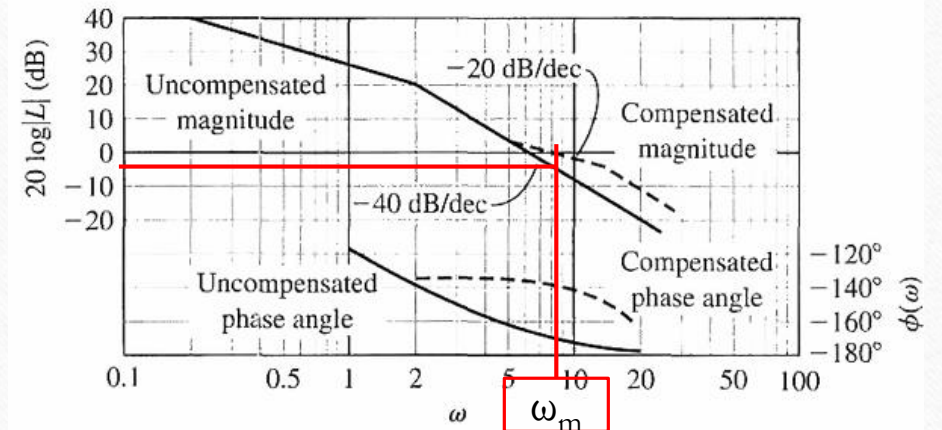
$$\frac{\alpha - 1}{\alpha + 1} = \sin 30^\circ = 0.5, \quad \text{and therefore } \alpha = 3.$$

The maximum phase lead occurs at  $\omega_m$ , and this frequency will be selected so that the new crossover frequency and  $\omega_m$  coincide. The magnitude of the lead network at  $\omega_m$  is  $10 \log \alpha = 10 \log 3 = 4.8 \text{ dB}$ . The compensated crossover frequency is then evaluated where the magnitude of  $G(j\omega)$  is  $-4.8 \text{ dB}$ , and thus  $\omega_m = \omega_c = 8.4$ . Drawing the compensated magnitude line so that it intersects the

0-dB axis at  $\omega = \omega_c = 8.4$ , we find that  $z = \omega_m / \sqrt{\alpha} = 4.8$  and  $p = \alpha z = 14.4$ . Therefore, the compensation network is

$$G_c(s) = \frac{1}{3} \frac{1 + s/4.8}{1 + s/14.4}$$

$$G_c(s) = \frac{1 + \alpha \tau s}{\alpha(1 + \tau s)}$$



# Phase-lead network

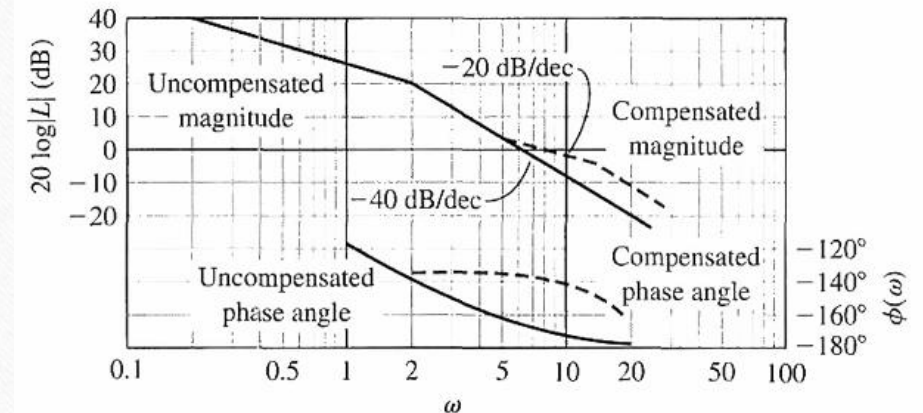
Determine the compensation network by completing the following steps:

The total DC loop gain must be raised by a factor of three in order to account for the factor  $1/\alpha = \frac{1}{3}$ . Then the compensated loop transfer function is

$$L(s) = G_c(s)G(s) = \frac{20(s/4.8 + 1)}{s(0.5s + 1)(s/14.4 + 1)} \quad (10.28)$$

To verify the final phase margin, we can evaluate the phase of  $G_c(j\omega)G(j\omega)$  at  $\omega = \omega_c = 8.4$  and thus obtain the phase margin. The phase angle is then

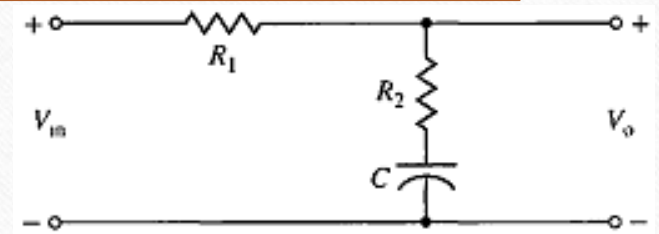
$$\begin{aligned} \phi(\omega_c) &= -90^\circ - \tan^{-1} 0.5\omega_c - \tan^{-1} \frac{\omega_c}{14.4} + \tan^{-1} \frac{\omega_c}{4.8} \\ &= -90^\circ - 76.5^\circ - 30.0^\circ + 60.2^\circ \\ &= -136.3^\circ. \end{aligned} \quad (10.29)$$





# Phase-lag network

- The transfer function of the **phase-lag** network is **RC network** suitable for compensating a feedback control



When  $t = R_2 C$  and  $\alpha = (R_1 + R_2) / R_2$ ,

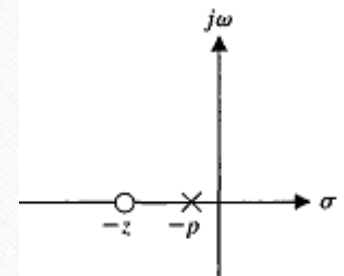
where  $z = 1/t$ ,  $p = 1/(\alpha t)$ .

We will now add the **integration-type** phase-lag network as a compensator and determine the compensated velocity constant.

$$G_c(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_2 + 1/(Cs)}{R_1 + R_2 + 1/(Cs)} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

$$G_c(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$

$$G_c(s) = \frac{1 + \tau s}{1 + \alpha\tau s} = \frac{1 s + z}{\alpha s + p}$$



The maximum phase lag occurs at  $\omega_m$

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}$$

# Phase-lag network

Determine the compensation network by completing the following steps:

1. **Obtain** the Bode diagram of the **uncompensated** system with the gain adjusted for the desired error constant.
2. **Determine** the phase margin of the **uncompensated** system and, if it is insufficient, proceed with the following steps.
3. **Determine** the frequency where the **phase margin** requirement would be satisfied if the magnitude curve crossed the **0-dB** line at this frequency,  $\omega'c$ . (Allow for  $5^\circ$  phase lag from the phase-lag network when determining the new crossover frequency.)
4. **Place** the zero of the compensator one decade below the new crossover frequency  $\omega'c$ , and thus ensure only  $5^\circ$  of additional phase lag at  $\omega'c$
5. **Measure** the necessary attenuation at  $\omega'c$  to ensure that the **magnitude** curve crosses at this frequency.
6. **Calculate**  $\alpha$  by noting that the attenuation introduced by the phase-lag network is  $-20 \log \alpha$  at  $\omega'c$ .
7. **Calculate** the pole as  $\omega_p = 1/(\alpha t) = \omega_z / \alpha$ , and the design is completed



# Phase-lead network

Determine the compensation network by completing the following steps:

A unity feedback control system has a loop transfer function

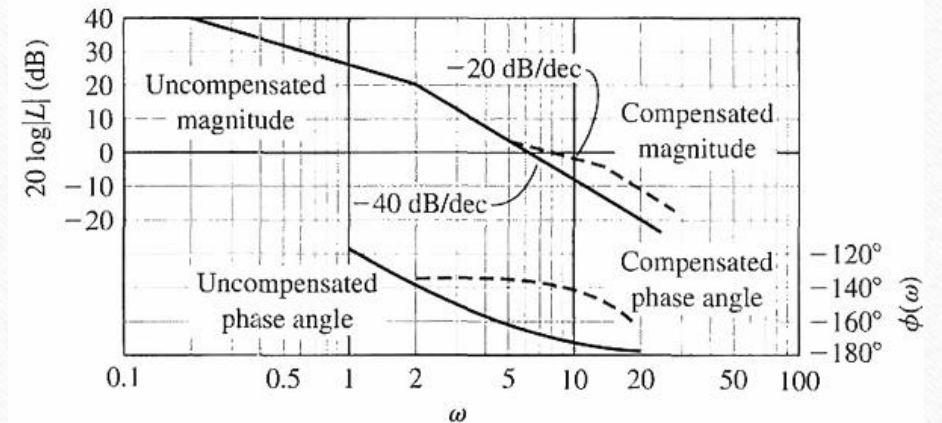
$$L(s) = \frac{K}{s(s+2)} \quad (10.21)$$

where  $L(s) = G_c(s)G(s)$  and  $H(s) = 1$ . We want to have a steady-state error for a ramp input equal to 5% of the velocity of the ramp. Therefore, we require that

$$K_v = \frac{A}{e_{ss}} = \frac{A}{0.05A} = 20. \quad (10.22)$$

Furthermore, we desire that the phase margin of the system be at least  $45^\circ$ . The first step is to plot the Bode diagram of the uncompensated transfer function

$$G(j\omega) = \frac{K_v}{j\omega(0.5j\omega + 1)} = \frac{20}{j\omega(0.5j\omega + 1)} \quad (10.23)$$



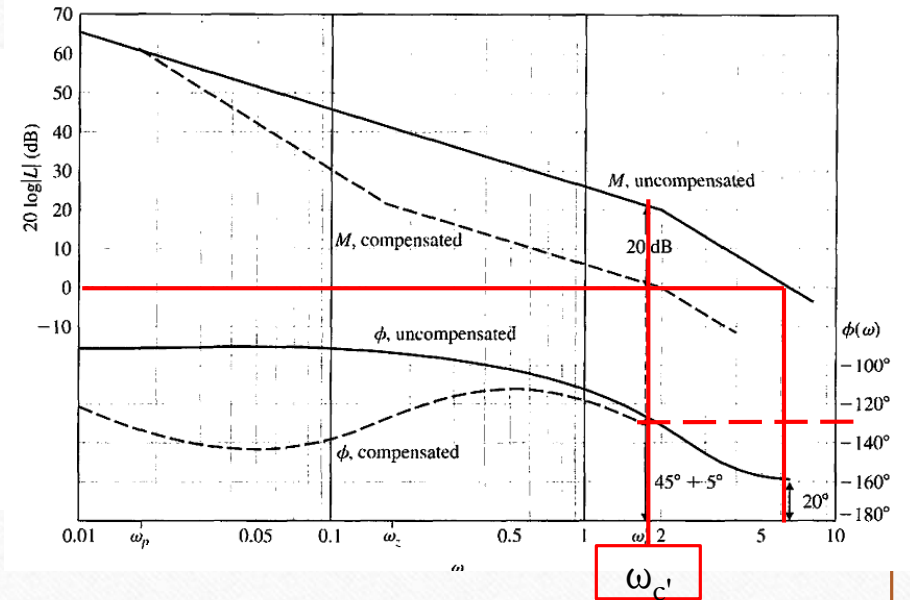
$$20 \log 20 = 26.02$$

at  $\omega = 1$

# Phase-lag network

Determine the compensation network by completing the following steps:

where  $K_v = K/2$ . We want  $K_v = 20$  while a phase margin of  $45^\circ$  is attained. The uncompensated Bode diagram is shown as a solid line in Figure 10.19. The uncompensated system has a phase margin of  $20^\circ$ , and the phase margin must be increased. Allowing  $5^\circ$  for the phase-lag compensator, we locate the frequency  $\omega$  where  $\phi(\omega) = -130^\circ$ , which is to be our new crossover frequency  $\omega'_c$ . In this case, we find that  $\omega'_c = 1.5$ , which allows for a small margin of safety. The attenuation necessary to cause  $\omega'_c$  to be the new crossover frequency is equal to 20 dB. Both the compensated and uncompensated magnitude curves are an asymptotic approximation. Both the actual curves are 2 dB lower than shown. Thus,  $\omega'_c = 1.5$ , and the required attenuation is 20 dB.





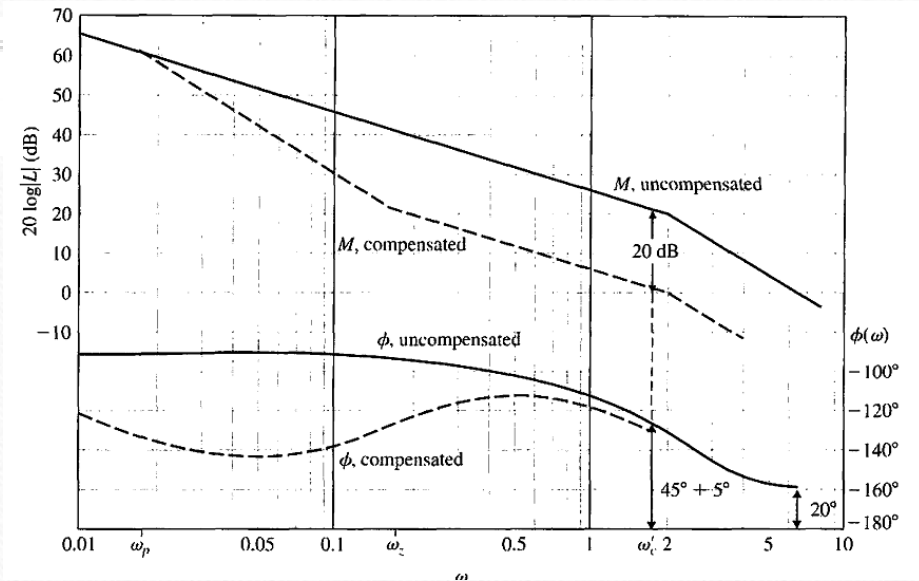
# Phase-lag network

Determine the compensation network by completing the following steps:

Then we find that  $20 \text{ dB} = 20 \log \alpha$ , or  $\alpha = 10$ . Therefore, the zero is one decade below the crossover, or  $\omega_z = \omega'_c/10 = 0.15$ , and the pole is at  $\omega_p = \omega_z/10 = 0.015$ . The compensated system is then

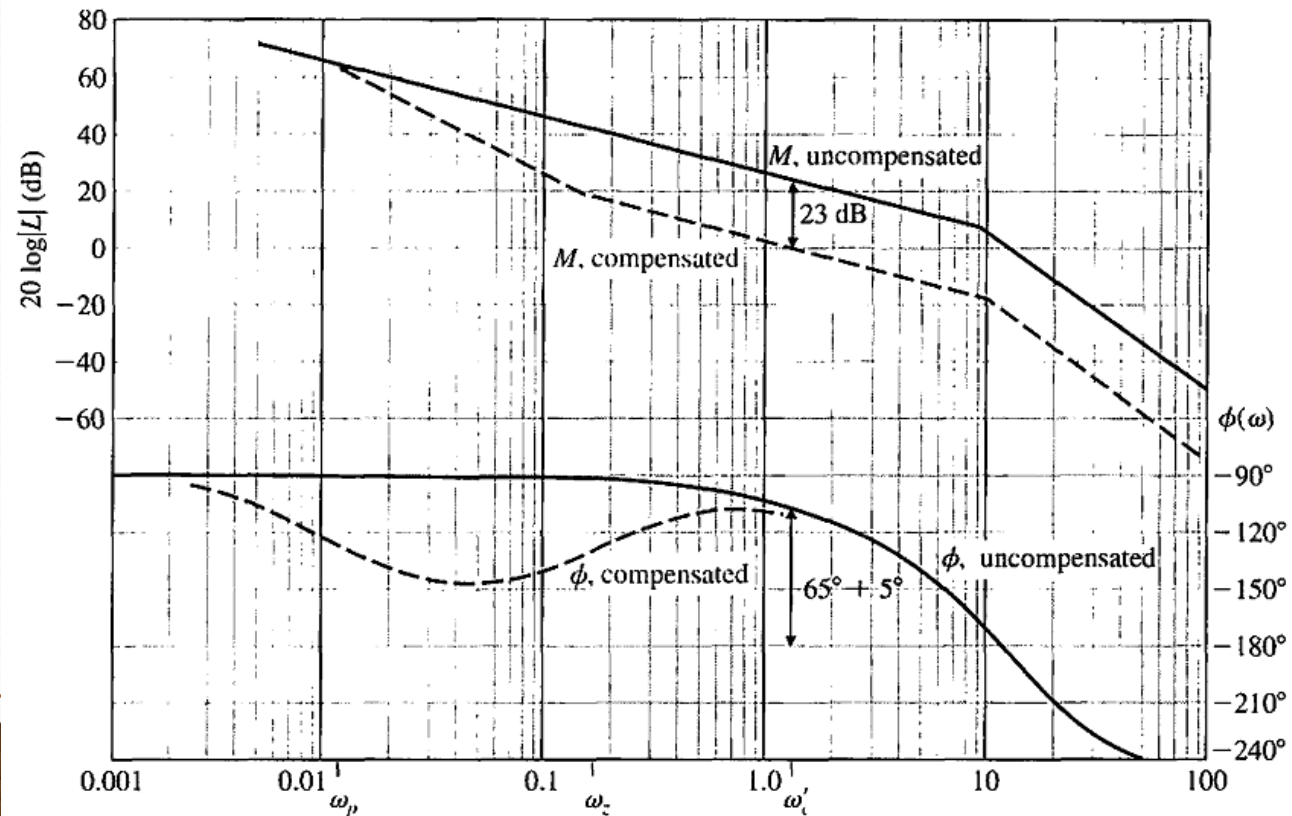
$$G_c(s) = \frac{1 + \tau s}{1 + \alpha \tau s} = \frac{1}{\alpha} \frac{s + z}{s + p}$$

$$G_c(j\omega)G(j\omega) = \frac{20(6.66j\omega + 1)}{j\omega(0.5j\omega + 1)(66.6j\omega + 1)} \quad (10.70)$$



# Phase-lag network

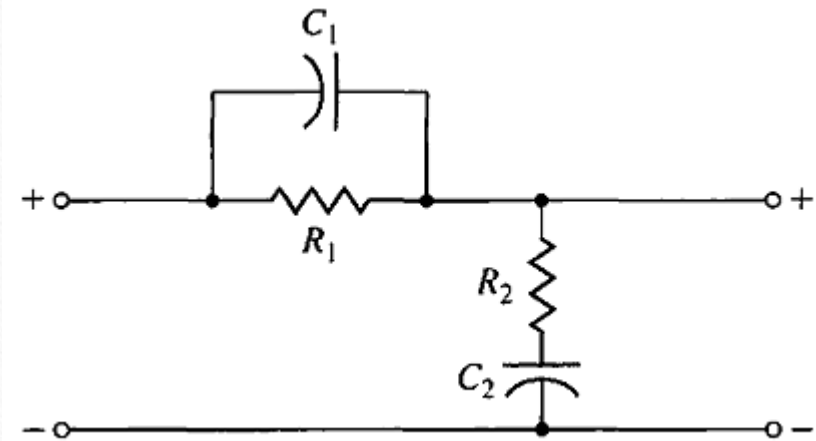
Determine the compensation network by completing the following steps:





# Lead-lag network

The phase-lead compensation network alters the frequency response of a network by adding a positive (leading) phase angle and therefore increases the phase margin at the crossover (0-dB) frequency. An analytical technique of selecting the parameters of a lead or lag network has been developed for the Bode diagram



$$\frac{V_2(s)}{V_1(s)} = \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2)s + 1}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{(1 + \alpha \tau_1 s)(1 + \beta \tau_2 s)}{(1 + \tau_1 s)(1 + \tau_2 s)}$$

# Lead-lag network

- If  $\alpha < 1$  yields a **lag** compensator and  $\alpha > 1$  yields a **lead** compensator.
- The **phase contribution** of the compensator at the design frequency  $\omega_c$  is

$$p = \tan \phi = \frac{\alpha\omega_c\tau - \omega_c\tau}{1 + (\omega_c\tau)^2\alpha}$$

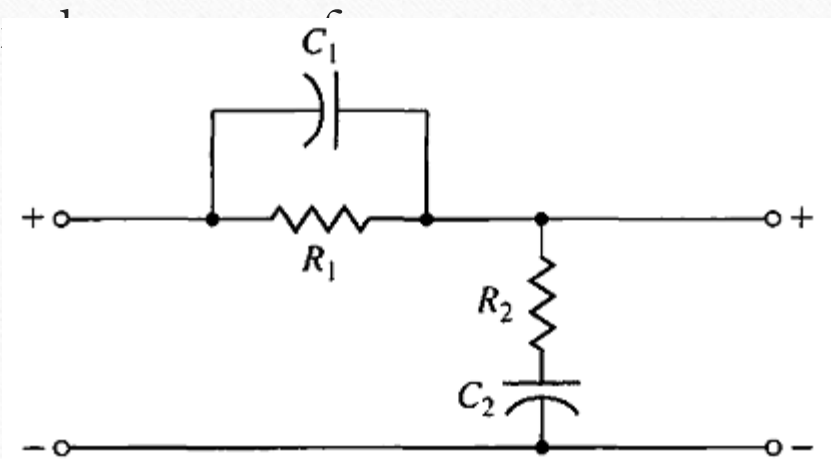
- The **magnitude**  $M$  (in dB) of the compensator at the design frequency  $\omega_c$  is

$$c = 10^{M/10} = \frac{1 + (\omega_c\alpha\tau)^2}{1 + (\omega_c\tau)^2}$$

- obtain the nontrivial solution equation for  $\alpha$  as

$$(p^2 - c + 1)\alpha^2 + 2p^2c\alpha + p^2c^2 + c^2 - c = 0.$$

- single-stage **compensator**, it is necessary that  $c > p^2 + 1$



$$\tau = \frac{1}{\omega_c} \sqrt{\frac{1-c}{c-\alpha^2}}$$



# Lead-lag network

Determine the compensation network by completing the following steps:

1. Select the desired  $\omega_c$ .
2. Determine the **phase** margin desired and therefore the required phase  $\varphi$ .

$$p = \tan \phi = \frac{\alpha\omega_c\tau - \omega_c\tau}{1 + (\omega_c\tau)^2\alpha}$$

3. Verify that the **phase lead** is applicable:  $\varphi > 0$  and  $M > 0$ .
4. Determine whether a single stage will be sufficient by testing  $c > p^2 + 1$ .
5. Determine  $\alpha$  from.  $(p^2 - c + 1)\alpha^2 + 2p^2c\alpha + p^2c^2 + c^2 - c = 0$ .

6. Determine  $\tau$  from. 
$$\tau = \frac{1}{\omega_c} \sqrt{\frac{1-c}{c-\alpha^2}}$$

If we need to design a single-lag compensator, then  $\varphi < 0$  and  $M < 0$  (step 3).

Step 4 will require  $c < 1/(1 + p^2)$ .